

Isospin Violation in $X(3872)$: Explanation From a New Tetraquark Model

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Abstract

New data for $X(3872)$ production in B decays provide a separation between X production and decay, sharpen several experimental puzzles and impose serious constraints on all models. Both charged and neutral B decays produce a narrow neutral resonant state that decays to both $J/\psi\rho$ and $J/\psi\omega$, while no charged resonances in the same multiplet are found. This suggests that the X is an isoscalar resonance whose production conserves isospin, while isospin is violated only in the decay by an electromagnetic interaction allowing the isospin-forbidden $J/\psi\rho$ decay. A tetraquark isoscalar X model is proposed which agrees with all present data, conserves isospin in its production and breaks isospin only in an electromagnetic $X(3872) \rightarrow J/\psi\rho^0$ decay. The narrow X decay width results from the tiny phase space available for the $J/\psi\omega$ decay and enables competition with the electromagnetic isospin-forbidden $J/\psi\rho$ decay which has much larger phase space. Experimental tests are proposed for this isospin production invariance.

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I. THE PUZZLE - HOW IS ISOSPIN BROKEN IN X(3872)?

A. Is isospin broken in production of X(3872) or in its decay?

The decays of the baffling resonance $X(3872)$ [1–6] have raised puzzles leading to suggestion that it breaks isospin symmetry [7,8]. In particular, both $X(3872) \rightarrow J/\psi\pi\pi$ and $X(3872) \rightarrow J/\psi\omega$ decay modes have been observed [9,10]. Since isospin symmetry has been shown to be an excellent symmetry of strong interactions and $SU(3)_{\text{flavor}}$ been shown to be a good approximate symmetry, we investigate ways to solve this puzzle.

New data on $B \rightarrow KX$ sharpen this puzzle. The weak B decay provides an unambiguous production mechanism. The dominant b quark decay into charmonium states goes via the vertex

$$b \rightarrow c\bar{c}s \quad (1)$$

Although the weak interaction generally violates isospin, this vertex conserves isospin. The b , c and s flavors are all isospin scalars. The decay of a B meson state $B(\bar{b}q)$ in which a \bar{b} antiquark is bound to a nonstrange quark of flavor q is a $\delta I = 0$ transition. If the strong interaction following the weak decay is invariant under the interchange of the u and d flavors, denoted by $\mathcal{I}(u \leftrightarrow d)$, we have $\mathcal{I}(u \leftrightarrow d)|B^+\rangle = |B^0\rangle$ and we find the transmission matrix elements $\langle f|T|B\rangle$ for any B decay into any final state denoted by $|f\rangle$ satisfy the relation

$$\langle f|T|B^+\rangle = \langle \mathcal{I}(u \leftrightarrow d)f|T|B^0\rangle \quad (2)$$

The new data violate isospin invariance, also called charge independence, which is a continuous $SU(2)$ transformation in the u, d space. But they seem to satisfy the isospin reflection (2) which is a discrete subgroup of $SU(2)$ also called charge symmetry and is less than charge independence. For example, the charge symmetry reflection relates the pair of decays $B^+ \rightarrow J/\psi K^+\rho^0$ and $B^0 \rightarrow J/\psi K^0\rho^0$ which can go via a neutral X . Charge symmetry also relates the pair $B^+ \rightarrow J/\psi K^0\rho^+$ and $B^0 \rightarrow J/\psi K^+\rho^-$ which cannot go via a neutral X . Full charge independence of isospin invariance relates all four decays and is violated by the absence of a charged X .

This imposes serious constraints on all models for the X which are not easily satisfied. None of the proposed models for the X can explain all the following three observations

1. The X decays into both into the isovector $J/\psi\rho^0$ and the isoscalar $J/\psi\omega$ [9,10].
2. No charged X partner decaying into the charged partner of $J/\psi\rho^0$ has been found [11].
3. The X is produced at comparable rate both in charged and neutral $B^+ \rightarrow K^+X_u$ and $B^0 \rightarrow K^0X_d$ decays [12], where the states denoted by X_u and X_d are either the same state or two states satisfying $X_u = \mathcal{I}(u \leftrightarrow d)X_d$.

None of the proposed models can explain how both charged and neutral B mesons can produce a neutral resonant state that decays to both $J/\psi\rho$ and $J/\psi\omega$, while no charged resonances in the same multiplet are found. The models which use a $D^0\bar{D}^{*0}$ “deuson” [7] contain no d flavored quarks and are not easily produced produced from a neutral B which

has a spectator d quark. That already says that there is a problem which none of the proposed models can solve.

We now see that the new data raise the question of whether the isospin breaking occurs in the production of the resonance, in the propagation in time of the resonance or in the decay process. In the remainder of this paper we will show that all the problems listed above can be solved in a model that assumes the X is isoscalar and has no charged partner, that its production and propagation in time all satisfy isospin invariance, and all the isospin breaking occurs only in its decays into the isovector $J/\psi \rho^0$ which can be electromagnetic.

B. A nonet structure for $c\bar{c}q\bar{q}$ states in b decays

A charmonium $c\bar{c}$ quark pair is an $SU(3)_{\text{flavor}}$ singlet whose strong interactions with u , d and s quarks are flavor independent. We therefore assume that the strong dynamics of a light quark system and its flavor structure will be negligibly changed by the addition of a charmonium pair. Thus we assume that the spectrum of a system of a light quark $q\bar{q}$ pair and a $c\bar{c}$ will have the nonet structure of the light quark symmetry with flavor symmetry broken only by the mass difference between the strange and nonstrange quarks.

B decays via the vertex (1) into charmonium and two light quark mesons denoted by M_1 and M_2 are therefore described by the diagrams

$$B(b\bar{q}) \rightarrow c\bar{c}s\bar{q} \rightarrow (c\bar{c})s\bar{q}q\bar{q} \rightarrow M_1(s\bar{q})(c\bar{c})M_2(q\bar{q}) \quad (3)$$

$$B(b\bar{q}) \rightarrow c\bar{c}s\bar{q} \rightarrow (c\bar{c})s\bar{q}q\bar{q} \rightarrow M_1(s\bar{q})(c\bar{c})M_2(q\bar{q}) \quad (4)$$

where the \bar{q} is the spectator antiquark and $(q\bar{q})$ denotes a flavor singlet quark-antiquark pair created by gluons. The diagram (4) where the $q\bar{q}$ pair appears in the same M_2 final meson violates the well established OZI selection rule [13–15] which forbids a quark-antiquark pair created by gluons to appear in the same final meson.

We are interested in decays of the form $B \rightarrow K X(3872) \rightarrow K J/\psi \rho/\omega$. In these decays M_2 is a light quark neutral vector meson and the only contributing diagrams (3) which do not violate the OZI rule are

$$B(b\bar{q}) \rightarrow M_1(s\bar{q})(c\bar{c})M_2(q\bar{q}) \rightarrow M_1(s\bar{q})X_{\mathbf{q}} \rightarrow M_1(s\bar{q})J/\psi V_{\mathbf{q}} \quad (5)$$

Where $V_{\mathbf{q}}$ denotes the linear combination of neutral vector meson states with the quark constituents $\mathbf{q}\bar{\mathbf{q}}$ and $X_{\mathbf{q}}$ denotes the linear combination of $X(3872)$ that decays into $J/\psi V_{\mathbf{q}}$.

$$V_u = \frac{\rho + \omega}{\sqrt{2}}; \quad V_d = \frac{\rho - \omega}{\sqrt{2}} \quad (6)$$

If two amplitudes related by the isospin reflection are equal,

$$A[B(b\bar{u}) \rightarrow M_1(s\bar{u})J/\psi V_u] = A[B(b\bar{d}) \rightarrow M_1(s\bar{d})J/\psi V_d] \quad (7)$$

this then leads to the basic puzzle: *a neutral final state requires that the only components of the $q\bar{q}$ pair created by gluons that contribute to the X production must have the same flavor q as the spectator quark.* For example, consider the decay

$$B^- \rightarrow K^- X(3872) \rightarrow K^- J/\psi \rho^0 \quad (8)$$

The B^- meson contains a spectator \bar{u} quark. The final state contains three light quarks, one being the original spectator and an additional $q\bar{q}$ pair made by gluons. Since gluons are isoscalar, the additional $q\bar{q}$ pair must be isoscalar and can not make a ρ^0 . Thus one of the two quarks in the final state ρ^0 must be the spectator \bar{u} quark. Since ρ^0 is neutral, the other quark in ρ^0 must be the u quark coming from gluons and the $q\bar{q}$ pair produced by gluons must therefore be a $u\bar{u}$. Similarly, in

$$B^0 \rightarrow K^0 X(3872) \rightarrow K^0 J/\psi \rho^0, \quad (9)$$

the gluons must only make a $d\bar{d}$ pair. This is puzzling, because components of the $q\bar{q}$ pair with the opposite nonstrange flavor should be produced equally by gluons, but these would produce charged states, for example

$$B^-(b\bar{u}) \rightarrow \bar{K}^0 J/\psi \rho^-(d\bar{u}). \quad (10)$$

But no such charged narrow states have been seen at the X mass.

However, before jumping hastily to conclusions about isospin violation, we note some remarkable kinematics at the X mass. The X mass is very close to the $J/\psi\omega$ threshold and may even be slightly below. If the X has isospin zero and a spin and parity that forbid its decay into two pseudoscalar mesons, the lowest final state allowed by isospin-conserving strong interactions is $J/\psi\omega$, which has very small phase space. This can account for the observed narrow width of the isoscalar X . In some models the X can have an isovector partner. The large width of the ρ and its large lower mass tail allow the isovector partner of the X to be well above the $J/\psi\rho$ threshold. The transitions (8), (9) and (10) will all have a very large phase space and be very wide for the isovector partner and unobservable against a continuum background. Only the transition

$$B^0 \rightarrow K^0 X(3872) \rightarrow K^0 J/\psi \omega \quad (11)$$

survives with a narrow width for strong decay. However, even for a purely isoscalar $X(3872)$ the transitions (8) and (9) can go via an electromagnetic isospin breaking interaction. Although electromagnetic decays are generally much weaker than strong decays, the large phase space enables these electromagnetic decays to compete with the allowed strong decay (11).

This enormous phase space difference must play a role in any model for the $X(3872)$. Models like the deuson model which have a mixed isospin state for the $X(3872)$ cannot easily explain why the large phase space for the $J/\psi\rho$ decay does not overwhelm the $J/\psi\omega$ decay. If the transition matrix elements are comparable for the two decays, the phase space difference will greatly enhance the $J/\psi\rho$ decay branching ratio above the small $J/\psi\omega$ branching ratio.

Outside the X mass region where phase space is not so different we can find relations that can be tested by experiment. OZI-violating transitions involve an isoscalar pair created by gluons which can only produce the isoscalar ω . We first neglect the OZI violating transition. In charged B decays the light neutral vector meson in the final state appears with the constituents $u\bar{u}$ which is not an isospin eigenstate, and in neutral B decays it appears with constituents $d\bar{d}$ which also is not an isospin eigenstate. Thus all neutral nonstrange vector

mesons produced in these decays must be equal linear combinations of two isospin eigenstates with isospin zero and one, like ρ^0 and ω .

We can include a small correction to this $\rho^0 - \omega$ equality resulting from possible OZI violations which add an extra ω amplitude, but will not affect the ρ amplitude.

For the case where M_1 is a pseudoscalar meson eq. (5) gives the relation

$$A[B^\pm \rightarrow J/\psi K^\pm \omega] = A[B^\pm \rightarrow J/\psi K^\pm \rho^0] \quad (12)$$

This is the analog of the first quark-model OZI relation [16],

$$A[K^- p \rightarrow \Lambda \omega] = A[K^- p \rightarrow \Lambda \rho^0] \quad (13)$$

The relation (12) follows from $U(3)$ flavor symmetry and holds for all regions of phase space where $U(3)$ symmetry is not broken.

Since isospin is a subgroup of $U(3)$, the isospin reflection of eq. (12) is equally valid and related to eq. (12),

$$A[B^\pm \rightarrow J/\psi K^\pm \omega] = A[B^0 \rightarrow J/\psi K^0 \omega] = A[B^0 \rightarrow J/\psi K^0 \rho^0] = A[B^\pm \rightarrow J/\psi K^\pm \rho^0] \quad (14)$$

In the $U(3)$ symmetry limit all nine $q_i \bar{q}_j$ mesons are degenerate for all three values of the flavors i and j : $i, j = u, d, s$. There are two commonly noted symmetry breaking mechanisms: (1) the quark mass differences; (2) the annihilation diagram where one flavor is annihilated into gluons and a pair of another flavor is created.

When the annihilation diagram is dominant, $U(3)$ is broken into $SU(3) \otimes U(1)$, as in the pseudoscalar nonet, where the $SU(3)$ singlet η' is separated from the pseudoscalar octet. The $U(3)$ symmetry breaking in the pseudoscalars is generally attributed to the anomaly which affects only the singlet.

In the vector mesons, the quark mass difference between strange and nonstrange quarks is dominant and $U(3)$ is broken into $U(2)_{ud} \otimes U(1)_s$. At this stage the ρ and ω are degenerate. A small annihilation diagram then splits the $U(2)$ quartet into an $SU(2)$ triplet ρ and an $SU(2)$ singlet ω . In electromagnetic interactions where the electric charge difference between the u and d quarks is more important than the isospin conserving strong interaction, isospin is broken and the $u\bar{u}$ and $d\bar{d}$ states are closer to experiment than the isospin eigenstates.

However, the equality (7) indicates a different mechanism from the ud mass difference for isospin breaking. That two quark-antiquark pairs with different nonstrange flavors should be produced very unequally from gluons would indicate isospin breaking. But this type of isospin breaking is not predicted by any of the proposed models for the X . The data indicate that “charge symmetry” $\mathcal{I}(u \leftrightarrow d)$ is still valid but “charge independence” is violated. The full isospin symmetry which involves continuous transformations in the ud flavor space is broken. But the 180 degree isospin rotations which simply interchange u and d flavors remains intact. This symmetry breaking cannot result from a ud quark mass difference which would break both charge symmetry as well as charge independence. The alternative symmetry breaking by an electromagnetic interaction is now preferred.

C. Attempts to use the ud mass difference to explain isospin breaking

If isospin is broken by the ud mass difference as in the “deuson” [7] model for the $X(3872)$ resonance, one peculiar feature of the deuson wave function allows the prediction (12) for

charged B decays even when isospin is broken. The $\{\bar{D}^o(\bar{c}u) D^{*o}(c\bar{u})\}$ wave function has no d quarks, nor \bar{d} antiquarks. This enables its decay via the meson state V_u and suppresses its decay via the meson state V_d . The state V_u is an equal mixture of ρ and ω . But since the OZI-allowed final states for B^o decays (3) have a \bar{d} antiquark, we see that the neutral B decays into the $X(3872)$ resonance are forbidden in the deuson model.

If isospin is conserved, both the charged and neutral B mesons should decay equally into the $X(3872)$ resonance. Any isospin breaking will destroy this equality. The deuson model completely suppresses production of X in decay of neutral B meson. Thus the recent observation of neutral B decays into the X raises serious problems for any of the proposed models for the $X(3872)$ resonance.

If both the B^+ and B^o go into both $J/\psi\rho$ and $J/\psi\omega$ via the X and the X has no charged state, there is a serious problem.

There are four relevant hadronizations of the final states $c\bar{s}\bar{c}q$ state produced in the dominant b quark decay $\bar{b} \rightarrow \bar{s}c\bar{c}$, followed by the creation of an additional isoscalar $q\bar{q}$ pair from gluons.

- (a) $B^+(\bar{b}u) \rightarrow c\bar{s}\bar{c}u \rightarrow (\bar{s}u) J/\psi(\bar{u}u)$
- (b) $B^+(\bar{b}u) \rightarrow c\bar{s}\bar{c}u \rightarrow (\bar{s}d) J/\psi(\bar{d}u)$
- (c) $B^o(\bar{b}d) \rightarrow c\bar{s}\bar{c}d \rightarrow (\bar{s}u) J/\psi(\bar{u}d)$
- (d) $B^o(\bar{b}d) \rightarrow c\bar{s}\bar{c}d \rightarrow (\bar{s}d) J/\psi(\bar{d}d)$

where the quarks in final state four-quark system have been grouped to obey the OZI rule. In the isospin symmetry limit, all four diagrams are equal.

Experiments show that diagrams (a) and (d) are both present and approximately equal, while diagrams (b) and (c) produce charged X states that are not seen. Diagrams (a) and (d) go into one another by the $u \leftrightarrow d$ interchange which is an isospin reflection.

The problem is to find a symmetry breaking mechanism which breaks isospin by suppressing (c) and (d), while keeping the isospin reflection that relates (a) and (d). The deuson model or any other model which suppresses the d quark contribution by the quark mass difference suppresses (d) while keeping (a). The quark mass difference cannot suppress (b) and (c) while keeping (a) and (d).

We now note how these conclusions are changed if diagrams that violate the OZI rule are included. Since these have the isoscalar $q\bar{q}$ appearing in the same final state, they cannot produce charged states. The only vector meson they can produce is the isoscalar ω . The amplitude for this transition can interfere with the much larger OZI-allowed amplitude that produces the ω but it cannot produce the ρ .

II. THE SOLUTION; AN ISOSCALAR TETRAQUARK WITH ELECTROMAGNETIC ISOSPIN BREAKING

One possible direction [17] for resolving this puzzle is to consider the $X(3872)$ to be an isoscalar state with its strong decay into $J/\psi\omega$ allowed, but suppressed by phase space because it is so near the threshold. This explains why the $X(3872)$ is so narrow. Experimental support for this conjecture comes from a recent BaBar analysis of $B^+ \rightarrow J/\psi\omega$ which finds

that the $\pi^+\pi^-\pi^0$ invariant mass peaks more than $2\Gamma_\omega$ below the central value 783 MeV of ω mass [10]. The $J/\psi\rho$ decay is isospin forbidden in strong interactions, but can occur via an electromagnetic transition. This decay can compete with the isospin allowed $J/\psi\omega$ decay because it has a much higher phase space. The ρ is so wide that the $J/\psi\rho$ decay amplitude goes far below the $J/\psi\omega$ threshold.

One possible model for the $X(3872)$ is a $c\bar{c}q\bar{q}$ tetraquark which is an isospin singlet. The analogous isotriplet $c\bar{c}q\bar{q}$ tetraquark can decay strongly into $J/\psi\rho$ with such a large width that it will never be seen.

Other models like the deuson model [7] note that the mass of the $X(3872)$ is close to the $D^{*+}D^-$ threshold and consider isospin breaking by the ud mass difference at this threshold. That the $D^{*+}D^-$ and $J/\psi\omega$ thresholds are so close has no simple known explanation. In the tetraquark model the $D^{*+}D^-$ threshold plays no role while the closeness of the $X(3872)$ to the $J/\psi\omega$ thresholds is crucial.

The tetraquark model for isosinglet $X(3872)$ also explains the approximately equal production from charged and neutral B decays. There is no simple relation between the $J/\psi\omega$ and $J/\psi\rho$ decays. They are roughly equal because one is allowed by strong interactions but has little phase space, while the other is forbidden and has much larger phase space. This picture is not contradicted by the data.

All molecular models are in trouble because no $D^*\bar{D}$ model can explain how isospin is broken in a way that conserves the $u \leftrightarrow d$ interchange. The main point here is the closeness of the X to the $J/\psi\omega$ threshold, not the $D^*\bar{D}$ threshold.

In this picture the only isospin breaking is in the electromagnetic transition $X \rightarrow J/\psi\rho$. Thus all isospin relations for X production should be valid and can be tested by experiment. Since the decay $b \rightarrow c\bar{c}s$ is a $\Delta I = 0$ transition the final state of a B decay into charmonium and a kaon decay must have the isospin 1/2 of the spectator nonstrange quark. This means that the B decays to $J/\psi K\rho$ are related by isospin Clebsch-Gordan coefficients to give the experimentally testable relation

$$\begin{aligned}
& A[B^- \rightarrow J/\psi \bar{K}^0 \rho^-] = \\
& = -\sqrt{2} \cdot A[B^- \rightarrow J/\psi K^- \rho^0] = \\
& = -\sqrt{2} \cdot A[\bar{B}^0 \rightarrow J/\psi \bar{K}^0 \rho^0] = \\
& = A[\bar{B}^0 \rightarrow J/\psi K^- \rho^+]
\end{aligned} \tag{15}$$

The mass for the $c\bar{c}u\bar{u}$ tetraquark has been estimated [18] to be only 4% larger than the mass of two D mesons in a wave function where the two quarks are coupled to a color sextet and the two antiquarks are coupled to a color antisextet. This suggests that such states with the color sextet-antisextet coupling should be included in all calculations for tetraquark states. One crucial feature of the sextet coupling in this tetraquark model is that the interactions within the quark pair and the antiquark pair are repulsive and are overcome by the four attractive quark-antiquark interactions. The mean distance between the two quarks or two antiquarks is larger than the mean quark-antiquark distances [19,20]. These color couplings and color-space correlations are not found in most other tetraquark models.

The sextet-antisextet state is found to have considerably a considerably lower mass [18] than the commonly used triplet-antitriplet diquark-antidiquark [21,22] models. This casts doubt on all tetraquark calculations for the $X(3872)$ resonance which neglect the color space correlations [21–23]. These neglect the basic physics seen in the experimental hadron mass

spectrum and its description by QCD motivated models [24–28] showing that the attractive $q\bar{q}$ interaction as observed in mesons is much stronger than the attractive qq interaction observed in baryons.

The color-space correlation contributions to the energy may well be more important than the color-magnetic energy neglected here which dominates other tetraquark model calculations [21–23].

The $cu\bar{c}\bar{u}$ tetraquark which contains charge-conjugate sextet-antisextet states have charge conjugation quantum numbers that are conserved in strong and electromagnetic decays. Starting with the spin quantum numbers, we define the notation where the spins of the quark and the antiquark are respectively denoted by s_q and $s_{\bar{q}}$ and the total spin is denoted by S . The $|s_q, s_{\bar{q}}; S\rangle$ states $|1, 1; 0\rangle$, $|1, 1; 2\rangle$ and $|0, 0; 0\rangle$ are all even under C and therefore also even under CP . The $|1, 1; 1\rangle$ state is odd under C and therefore also odd under CP , while the $|0, 1; 1\rangle$ and $|1, 0; 1\rangle$ are linear combinations of even and odd C and the sum and difference of these states are respectively even and odd under both C and CP .

In order to make contact with experiment, we recall that observed decay modes of $X(3872)$ are $J/\psi\rho^0$, $J/\psi\omega$, $J/\psi\gamma$ and $\bar{D}D^*$. The mode which is conspicuously absent is $\bar{D}D$, which has a significantly larger phase space than both $J/\psi\rho^0$ and $J/\psi\omega$. In order to understand this pattern, we need to examine the constraints from the C and CP quantum numbers.

The $J/\psi\rho^0$, $J/\psi\omega$ and $J\psi\gamma$ are all even under C and even under CP (in S -wave). The only tetraquark state that is even under C and cannot decay into $D\bar{D}$ is the even C linear combination of $|0, 1; 1\rangle$ and $|1, 0; 1\rangle$. The $J^P = 1^+$, even- C linear combination of $|0, 1; 1\rangle$ and $|1, 0; 1\rangle$ can decay into $J/\psi\rho^0$ and $J/\psi\omega$ in an S -wave, and cannot decay into $D\bar{D}$, since a 1^+ state cannot go into two pseudoscalars. This $J^{PC} = 1^{++}$ tetraquark model agrees with all the published data for the $X(3872)$.

III. CONCLUSION

The apparent isospin violation in $X(3872)$ decays is explained by an isoscalar tetraquark model which conserves isospin, with the only isospin breaking arising from the electromagnetic transition to $J/\psi\rho$. This decay is isospin forbidden for strong interactions, but has much higher phase space than $J/\psi\omega$ because the ρ is so wide and the $J/\psi\rho$ decay amplitude goes far beyond the $J/\psi\omega$ threshold. Models which break isospin by the ud quark mass differences are unable to explain the data.

ACKNOWLEDGEMENTS

This work was supported by the U.S. Department of Energy, Office of Nuclear Physics, under contract DE-AC02-06CH11257. We are grateful to Barry Wicklund for pointing out the possibility of an isospin violating EM decay of the $X(3872)$. We also thank Arafat Mokhtar for interesting discussions of the data.

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